$$\max \ f(x,y) = x^3 + 2xy$$

$$\text{s.t.} \quad x \geq 6 \quad (1)$$

$$y \leq 17 \quad (2)$$

$$x^2 + y^2 \leq 1000 \quad (3)$$

Step 1: NDCQ

$$J = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 2x & 2y \end{pmatrix}$$

[illegible]

Constraint $\rightarrow$ binding (works as equality)

$\rightarrow$ non-binding (otherwise)

Consider J just for binding constraints:

Binding constraint(s)	J	Rank
(1) $x=6$	$\begin{pmatrix} 1 & 0 \end{pmatrix}$	full
(2) $y=17$	$\begin{pmatrix} 0 & 1 \end{pmatrix}$	full
(3) $y^2 = 1000 - x^2$	$\begin{pmatrix} 2x & 2y \end{pmatrix} = \begin{pmatrix} 2x & \pm 2\sqrt{1000-x^2} \end{pmatrix}$	full
(1) (2) $x=6$, $y=17$	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	full
(2) (3) $y=17$ $x=\pm\sqrt{1000-17^2}$	$\begin{pmatrix} 0 & 1 \\ \pm 2\sqrt{1000-17^2} & 34 \end{pmatrix}$	full
(1) (3) $x=6$ $y=\pm\sqrt{964}$	$\begin{pmatrix} 1 & 0 \\ 12 & \pm 2\sqrt{964} \end{pmatrix}$	full
(1) (2) (3) $x=6$, $y=17$ $6^2+17^2 \neq 1000$	$\emptyset$	$\emptyset$

Rank is always full, So, NDCQ always holds.

Step 2. Since $f \to \max$, all constraints must be "$\leq$".

$$-x \leq -6$$
$$y \leq 17$$
$$x^2 + y^2 \leq 1000$$

Step 3.

$$\mathcal{L} = x^3 + 2xy + \lambda_1 (1000 - x^2 - y^2) + \lambda_2 (17 - y) +$$
$$+ \lambda_3 (x - 6)$$

Step 4. FOC:

$$\begin{cases} \mathcal{L}'_x = 3x^2 + 2y - 2x\lambda_1 + \lambda_3 = 0 \\ \mathcal{L}'_y = 2x - 2y\lambda_1 - \lambda_2 = 0 \\ \lambda_1 \mathcal{L}'_{\lambda_1} = 0 \\ \lambda_2 \mathcal{L}'_{\lambda_2} = 0 \\ \lambda_3 \mathcal{L}'_{\lambda_3} = 0 \\ \lambda_3, \lambda_2, \lambda_3 \geq 0 \end{cases}$$

Solve it.